

JEE(Main) : LEADER TEST SERIES / JOINT PACKAGE COURSE

Test Type : Major Test

ANSWER KEY**PART-1 PHYSICS**

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	A	C	C	C	B	C	D	A	D
	Q.	11	12	13	14	15	16	17	18	19	20
SECTION-II	A.	D	C	D	D	A	A	B	C	D	A
	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-II	A.	1.60	500.00	1.70	6.80	0.50	26.00	60.00	7.14	12.00	396.00

PART-2 CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	B	D	D	D	C	C	C	C	D
	Q.	11	12	13	14	15	16	17	18	19	20
SECTION-II	A.	C	B	B	B	D	B	C	B	B	A
	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-II	A.	4.00	3.00	2.00	6.00	8.00	45.00	500.00	100.00	250.00	3.00

PART-3 MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	C	B	A	C	B	D	B	D	C
	Q.	11	12	13	14	15	16	17	18	19	20
SECTION-II	A.	D	C	D	C	B	C	B	D	B	C
	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-II	A.	10.00	18.00	2160.00	17.00	1.00	7.00	1.16	5.00	2.00	108.00

HINT - SHEET**PART-1 : PHYSICS****SECTION-I**

1. Reynolds Number = $\frac{\rho v d}{\eta}$

Volume flow rate = $v \times \pi r^2$

$$v = \frac{100 \times 10^{-3}}{60} \times \frac{1}{\pi \times 25 \times 10^{-4}}$$

$$v = \frac{2}{3\pi} \text{ m/s}$$

$$\text{Reynolds Number} = \frac{10^3 \times 2 \times 10 \times 10^{-2}}{10^{-3} \times 3\pi} \approx 2 \times 10^4$$

order 10^4

2. $-ms \frac{dT}{dt} = e\sigma A (T^4 - T_0^4)$

$$-\frac{dT}{dt} = \frac{e\sigma A}{ms} (T^4 - T_0^4)$$

$$-\frac{dT}{dt} = \frac{4e\sigma AT_0^3}{ms} (\Delta T)$$

$$T = T_0 + (T_i - T_0)e^{-kt}$$

$$\text{where } k = \frac{4e\sigma AT_0^3}{ms}$$

$$k = \frac{4e\sigma AT_0^3}{\rho v s}$$

$$\left| \frac{dT}{dt} \right| \propto k$$

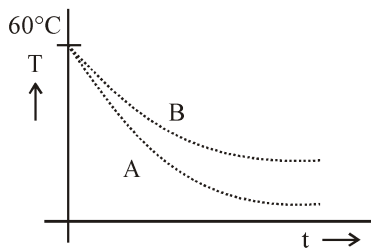
$$\therefore \left| \frac{dT}{dt} \right| \propto \frac{1}{\rho s}$$

$$\rho_A s_A = 2000 \times 8 \times 10^2 = 16 \times 10^5$$

$$\rho_B s_B = 4000 \times 10^3 = 4 \times 10^6$$

$$\rho_A s_A < \rho_B s_B$$

$$\left| \frac{dT}{dt} \right|_A > \left| \frac{dT}{dt} \right|_B$$



3. Energy of catapult = $\frac{1}{2} \times \left(\frac{\Delta \ell}{\ell} \right)^2 \times Y \times A \times \ell$

= Kinetic energy of the ball = $\frac{1}{2} m v^2$

therefore,

$$\frac{1}{2} \times \left(\frac{20}{42} \right)^2 \times Y \times \pi \times 3^2 \times 10^{-6} \times 42 \times 10^{-2}$$

$$= \frac{1}{2} \times 2 \times 10^{-2} \times (20)^2$$

$Y \approx 3 \times 10^6 \text{ Nm}^{-2}$

4. Suppose 'm' gram of water evaporates then, heat required

$\Delta Q_{\text{req}} = mL_v$

Mass that converts into ice = (150 - m)

So, heat released in this process

$\Delta Q_{\text{rel}} = (150 - m) L_f$

Now,

$\Delta Q_{\text{rel}} = \Delta Q_{\text{req}}$

$(150 - m) L_f = mL_v$

$m(L_f + L_v) = 150 L_f$

$m = \frac{150 L_f}{L_f + L_v}$

$m = 20\text{g}$

5. $v_{\text{rms}} = \sqrt{\frac{3RT}{m}}$ $v_{\text{escape}} = \sqrt{2gR_e}$

$v_{\text{rms}} = v_{\text{escape}}$

$\frac{3RT}{m} = 2gR_e$

$\frac{3 \times 1.38 \times 10^{-23} \times 6.02 \times 10^{26}}{2} \times T$

$= 2 \times 10 \times 6.4 \times 10^6$

$T = \frac{4 \times 10 \times 6.4 \times 10^6}{3 \times 1.38 \times 6.02 \times 10^3} = 10 \times 10^3 = 10^4 \text{K}$

Note : Question gives avogadro Number $N_A = 6.02 \times 10^{26} / \text{kg}$ but we take $N_A = 6.02 \times 10^{26} / \text{kmol}$.

6. $A = A_0 e^{-\gamma t}$

$A = \frac{A_0}{2}$ after 10 oscillations

\therefore After 2 seconds

$\frac{A_0}{2} = A_0 e^{-\gamma(2)}$

$2 = e^{2\gamma}$

$\ln 2 = 2\gamma$

$\gamma = \frac{\ln 2}{2}$

$\therefore A = A_0 e^{-\gamma t}$

$\ln \frac{A_0}{A} = \gamma t$

$\ln 1000 = \frac{\ln 2}{2} t$

$2 \left(\frac{3 \ln 10}{\ln 2} \right) = t$

$\frac{6 \ln 10}{\ln 2} = t$

$t = 19.931 \text{ sec}$

$t \approx 20 \text{ sec}$

7. Angular impulse = change in angular momentum

$\tau \Delta t = \Delta L$

$mg \frac{\ell}{2} \times .01 = \frac{m \ell^2}{3} \omega$

$\omega = \frac{3g \times 0.01}{2\ell}$

$= \frac{3 \times 10 \times .01}{2 \times 0.3}$

$= \frac{1}{2} = 0.5 \text{ rad/s}$

time taken by rod to hit the ground

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5}{10}} = 1 \text{ sec.}$

in this time angle rotate by rod

$\theta = \omega t = 0.5 \times 1 = 0.5 \text{ radian}$

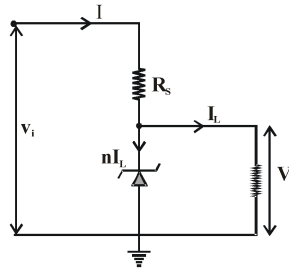
8. Particles can have same magnitude of momentum but different directions.

9. $N_t = N_0 e^{-\lambda t}$
 $\lambda \ln N_t = -\lambda t + \ln N_0$
 at $t = 0$, $\ln N_t = 3 = \ln N_0$, $N_0 = e^3 = (2.7)^3 = 20$
 at $t = 60s$, $\ln N_t = -\lambda \times 60 + 3 = 0$

$$\lambda = \frac{3}{20} = \frac{1}{20} = 0.05/s$$

$$N_t = N_0 e^{-\lambda t} = 20 e^{-0.05t/s}$$

10. $V_i - V_L = IR_S$
 $V_i - V_L = (nI_L + I_L)R_S$
 $R_S = \frac{V_i - V_L}{(n+1)I_L}$



11. At time t
 $\frac{N_B}{N_A} = .3 \Rightarrow N_B = .3 N_A$

also let initially there are total N_0 number of nuclei

$$N_A + N_B = N_0$$

$$N_A = \frac{N_0}{1.3}$$

Also as we know

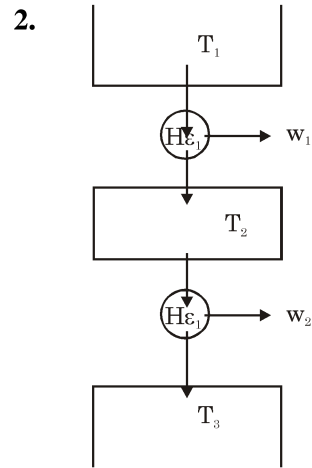
$$N_A = N_0 e^{-\lambda t} ; \frac{N_0}{1.3} = N_0 e^{-\lambda t}$$

$$\frac{1}{1.3} = e^{-\lambda t} \Rightarrow \ln(1.3) = \lambda t \text{ or } t = \frac{\ln(1.3)}{\lambda}$$

$$t = \frac{\ln(1.3)}{\frac{\ln(2)}{T}} = \frac{\ln(1.3)}{\ln(2)} T$$

SECTION-II

1. $M = \int_0^R \rho_0 r (2\pi r dr) = \frac{\rho_0 \times 2\pi \times R^3}{3}$
 $I_0 = \int_0^R \rho_0 r (2\pi r dr) \times r^2 = \frac{\rho_0 \times 2\pi R^5}{5}$
 (MOI about COM)
 by parallel axis theorem
 $I = I_0 + MR^2$
 $= \frac{\rho_0 \times 2\pi R^5}{5} + \frac{\rho_0 \times 2\pi R^3}{3} \times R^2 = \rho_0 2\pi R^5 \times \frac{8}{15}$
 $= MR^2 \times \frac{8}{5}$



2. $w_1 = w_2$
 $\Delta u_1 = \Delta u_2$
 $T_3 - T_2 = T_2 - T_1$
 $2T_2 = T_1 + T_3$
 $T_2 = 500 \text{ K}$

3. Given
 $\frac{Y_A}{Y_B} = \frac{7}{4}$ $L_A = 2\text{m}$ $A_A = \pi R^2$
 $L_B = 1.5\text{m}$ $A_B = \pi(2\text{mm})^2$

$$\frac{F}{A} = Y \left(\frac{\ell}{L} \right)$$

given F and ℓ are same $\Rightarrow \frac{AY}{L}$ is same

$$\frac{A_A Y_A}{L_A} = \frac{A_B Y_B}{L_B}$$

$$\Rightarrow \frac{(\pi R^2) \left(\frac{7}{4} Y_B \right)}{2} = \frac{\pi (2\text{mm})^2 \cdot Y_B}{1.5}$$

$$R = 1.74 \text{ mm}$$

4. $T = \frac{30 \text{ sec}}{20}$ $\Delta T = \frac{1}{20} \text{ sec.}$
 $L = 55 \text{ cm}$ $\Delta L = 1\text{mm} = 0.1 \text{ cm}$
 $g = \frac{4\pi^2 L}{T^2}$

percentage error in g is

$$\frac{\Delta g}{g} \times 100\% = \left(\frac{\Delta L}{L} + \frac{2\Delta T}{T} \right) 100\%$$

$$= \left(\frac{0.1}{55} + \frac{2 \left(\frac{1}{20} \right)}{\frac{30}{20}} \right) 100\% \approx 6.8\%$$

5. $\frac{hc}{\lambda} = W_0 + \frac{1}{2}mv_{\max}^2$

Assuming W_0 to be negligible in comparison to $\frac{hc}{\lambda}$

i.e. $v_{\max}^2 \propto \frac{1}{\lambda} \Rightarrow v_{\max} \propto \frac{1}{\sqrt{\lambda}}$

(On increasing wavelength λ to 4λ , v_{\max} becomes half).

6. $\frac{r_n}{r_{He}} = \left(\frac{A}{4}\right)^{1/3} = (14)^{1/3}$

$\Rightarrow A = 56$

Given $N = 30$

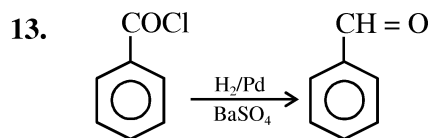
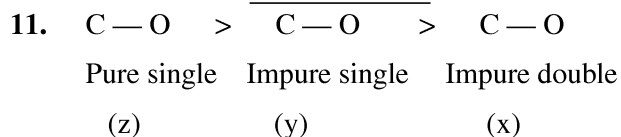
$\therefore Z = A - N = 56 - 30 = 26$

10. $n \propto \frac{1}{\ell} \Rightarrow n_1 \ell_1 = n_2 \ell_2 \Rightarrow (n+4)49 = (n-4)50$

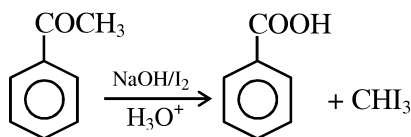
$\Rightarrow n = 396$

PART-2 : CHEMISTRY

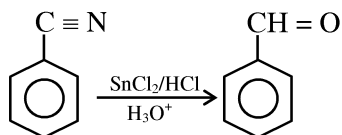
SECTION-I



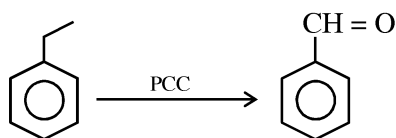
Rosenmund reduction



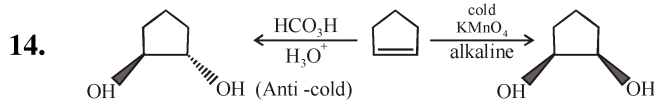
Iodoform reaction



Stephenson reaction



Mild oxidation



16. $N_2O_4 \rightleftharpoons 2NO_2$
 $1 \qquad \qquad 0$
 $1-x \qquad \qquad 2x = 1+x$
 $= 1-\alpha \qquad \qquad 2\alpha = 1+\alpha$

$k_p = \frac{\left(\frac{2\alpha}{1+\alpha} \times p\right)^2}{\left(\frac{1-\alpha}{1+\alpha} \times p\right)}$

$k_p = \frac{4\alpha^2 \cdot p}{1-\alpha^2}$
 $K_p - K_p \alpha^2 = 4k^2 \cdot p$
 $\alpha^2 (4p + k_p) = K_p$

$\alpha = \sqrt{\frac{k_p}{4p + k_p}}$

17. $\frac{\Delta T_b}{\Delta T_f} = \frac{k_b \times m}{k_f \times m}$
 $\frac{1}{\Delta T_f} = \frac{0.52 \times m}{1.86 \times m}$

$\Delta T_f = \frac{1.86}{0.52} = 3.61^\circ C$

$\therefore (F.P)_{\text{solution}} = -3.61 C$

18. $\Delta H = \Delta U + \Delta ngRT$
 $-3271 = \Delta U + \frac{(-1.5) \times 8.314 \times 300}{1000}$

$\Delta U = -3267.25 \text{ KJ/mol}$

for 1 mole of benzene

ΔU for 1.5 mol of benzene

$= -3267.25 \times 1.5$

$= -4900.25 \text{ KJ}$

19. $pH = pK_a + \log \frac{1/3}{2/3}$

20. $pH = pK_a + \log \frac{[HCO_3^-]}{[H_2CO_3]}$

or $7.4 = 6.1 + \log \frac{[HCO_3^-]}{[H_2CO_3]}$

or $\frac{[HCO_3^-]}{[H_2CO_3]} = 20$

SECTION-II

7. $t_{x\%} = \frac{2.303}{K} \log \frac{100}{100-x}$
 $= \frac{2.303 t_{1/2}}{\ln 2} \log \frac{100}{(100-99.999)}$
 $= \frac{2.303 \times t_{1/2}}{2.303 \log 2} \log \frac{100}{10^{-3}}$
 $\frac{30}{0.3} \log 10^5 = \frac{30 \times 5}{0.3} = 500$

8. $\frac{P - P_s}{P} = x_{\text{solute}}$
 $\frac{P - 80}{P} = 0.2$

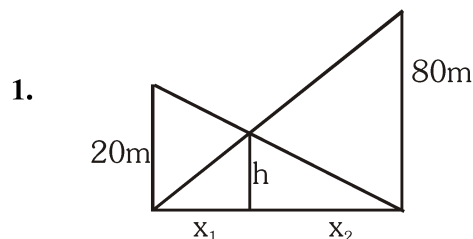
$P = 100 \text{ torr.}$

9. $nC_v(T_2 - T_1) = -75 \text{ cal}$
 $0.1 \times \frac{3R}{2} (T_2 - T_1) = -75$
 $T_2 - T_1 = \frac{-75}{0.3}$
 $T_2 - 500 = -250$
 $T_2 = 250 \text{ K.}$

10. $[H^+] = C\alpha = 0.1 \times \frac{1}{100} = 10^{-3} \text{ M}$
 $\text{pH} = -\log [H^+] = -\log 10^{-3} = 3$

PART-3 : MATHEMATICS

SECTION-I



by similar triangle

$\frac{h}{x_1} = \frac{80}{x_1 + x_2} \dots(1)$

by $\frac{h}{x_2} = \frac{20}{x_1 + x_2} \dots(2)$

by (1) and (2)

$\frac{x_2}{x_1} = 4 \text{ or } x_2 = 4x_1$

$\Rightarrow \frac{h}{x_1} = \frac{80}{5x_1}$

or $h = 16 \text{ m}$

2. $(p \rightarrow q) \wedge (q \rightarrow \sim p)$
 $\equiv (\sim p \vee q) \wedge (\sim q \vee \sim p)$
 $\equiv \sim p \vee (q \wedge \sim q)$
 $\equiv \sim p \vee C \equiv \sim p$

3. $|M| \neq 0$ [M is invertible]
 $\text{adj}(\text{adj}M) = |M| M = M^{-1} \dots(1)$
 $\Rightarrow |M|MM = M \cdot M^{-1}$
 $\Rightarrow |M| \cdot M^2 = I$
 $\Rightarrow |M| |M^2| = I$
 $\Rightarrow |M|^5 = |I| \Rightarrow |M| = 1$

From

(1) $M^{-1} = M$

$\Rightarrow I = M^2$

$\text{adj}(M^2) = \text{adj}(I) = I$

4. $\frac{2p}{3} = \frac{3q}{5} = \frac{4r}{7} = k$

$p = \frac{3k}{2}, q = \frac{5k}{3}, r = \frac{7k}{4}$

$p : q : r = \frac{3k}{2} : \frac{5k}{3} : \frac{7k}{4}$

$= 18 : 20 : 21$

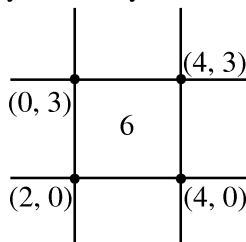
6. $2x^3 + 5x^2 + 2x - 1 = (x + 1)(2x^2 + 3x - 1)$

\Rightarrow either $x = -1$ is root of equation

or $x^2 + ax + b = 0$ & $2x^2 + 3x - 1 = 0$ have both roots in common.

7. $x \geq 2, x \leq 4,$

$y \geq 2, y \leq 3$



9. $|f(x)| = \begin{cases} 1 & , -2 \leq x < 0 \\ 1 - x^2 & , 0 \leq x < 1 \\ x^2 - 1 & , 1 \leq x \leq 2 \end{cases}$

and $f(|x|) = x^2 - 1, x \in [-2, 2]$

Hence $g(x) = \begin{cases} x^2 & , x \in [-2, 0) \\ 0 & , x \in [0, 1) \\ 2(x^2 - 1) & , x \in [1, 2] \end{cases}$

It is not differentiable at $x = 1$

10. Differentiating with respect to x,
 $x \cdot \frac{1}{\ln x} \cdot \frac{1}{x} + \ln(\ln x) - 2x + 2y \cdot \frac{dy}{dx} = 0$

at x = e we get

$$1 - 2e + 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{2e - 1}{2y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2e - 1}{2\sqrt{4 + e^2}} \text{ as } y(e) = \sqrt{4 + e^2}$$

11. $y = x^2 - 5x + 5$
 $\frac{dy}{dx} = 2x - 5 = 2 \Rightarrow x = \frac{7}{2}$

at $x = \frac{7}{2}, y = \frac{-1}{4}$

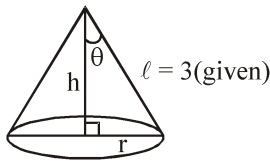
Equation of tangent at $(\frac{7}{2}, \frac{-1}{4})$ is

$$2x - y - \frac{29}{4} = 0$$

Now check options

$$x = \frac{1}{8}, y = -7$$

12.



$$\therefore h = 3 \cos \theta$$

$$r = 3 \sin \theta$$

Now,

$$V = \frac{1}{3} \pi r^2 h = \frac{\pi}{3} (9 \sin^2 \theta) \cdot (3 \cos \theta)$$

$$\therefore \frac{dV}{d\theta} = 0 \Rightarrow \sin \theta = \sqrt{\frac{2}{3}}$$

Also, $\left. \frac{d^2V}{d\theta^2} \right|_{\sin \theta = \sqrt{\frac{2}{3}}} = \text{negative}$

\Rightarrow Volume is maximum,

when $\sin \theta = \sqrt{\frac{2}{3}}$

$$\therefore V_{\max} \left(\sin \theta = \sqrt{\frac{2}{3}} \right) = 2\sqrt{3}\pi \text{ (in cu. m)}$$

13. Let $x^{1/6} = t$

$$\int \left(\frac{t^6 + t^4 + t}{t^6(1+t^2)} \right) 6t^5 dt = 6 \left(\tan^{-1} t + \frac{t^4}{4} \right) + c$$

14. $I = \int_{-\frac{1}{8}}^{\frac{1}{8}} \cos^{-1}(x^5 + 5x) dx$

$$2I = \int_{-\frac{1}{8}}^{\frac{1}{8}} \pi dx \quad \therefore I = \frac{\pi}{8}$$

15. $\int \frac{y dy}{1+y^2} = \int \frac{1+x+x^2}{x(1+x^2)} dx$

$$\frac{1}{2} \ln(1+y^2) = \tan^{-1} x + \ln x + c$$

$$\Rightarrow \ln \left(\frac{1+y^2}{x^2} \right) = 2 \tan^{-1} x + c$$

when $x = 1, y = 0 \therefore c = -\pi/2$

SECTION-II

1. $\tan \alpha + \tan \beta = \frac{\lambda\sqrt{2}}{k+1}$

$$\tan \alpha \cdot \tan \beta = \frac{k-1}{k+1}$$

$$\tan(\alpha + \beta) = \frac{\frac{\lambda\sqrt{2}}{k+1}}{1 - \frac{k-1}{k+1}} = \frac{\lambda\sqrt{2}}{2} = \frac{\lambda}{\sqrt{2}}$$

$$\Rightarrow \frac{\lambda^2}{2} = 50 \Rightarrow \lambda = 10 \text{ \& } -10$$

2. Variance of first 'n' natural numbers
 $= \frac{n^2 - 1}{12} = 10 \Rightarrow n = 11$

and variance of first 'm' even natural numbers

$$= 4 \left(\frac{m^2 - 1}{12} \right) \Rightarrow \frac{m^2 - 1}{3} = 16 \Rightarrow m = 7$$

$$m + n = 18$$

4. $x + y + z = 10$

Total no. of non negative sol. = $10 + 3 - 1$

For C_{3-1}

No. of cases in which z is even = ${}^{12}C_2 = 66$

Let $z = 2K$

$$x + y = 10 - 2K$$

$$x, y \geq 0$$

No. of solution = $10 - 2K + 2 - 1$

$$C_{2-1}$$

$$= 11 - 2K$$

Now

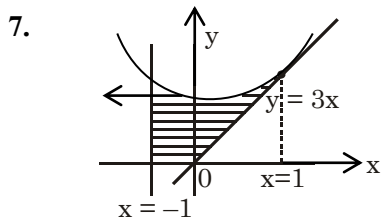
$$11 - 2K = 11 + 9 + 7 + 5 + 3 + 1 = 36$$

$$\text{Prof} = \frac{36}{66} = \frac{6}{11} = \frac{p}{q}$$

$$p + q = 17$$

5.
$$\lim_{x \rightarrow 0} \frac{x \tan^2 2x}{\tan 4x \sin^2 x} = \lim_{x \rightarrow 0} \frac{x \left(\frac{\tan^2 2x}{4x^2} \right) 4x^2}{\left(\frac{\tan 4x}{4x} \right) 4x \left(\frac{\sin^2 x}{x^2} \right) x^2} = 1$$

6. $f(x) = 3x^2 - 6(a - 2)x + 3a$
 $f(x) \geq 0 \forall x \in (0, 1]$
 $f(x) \leq 0 \forall x \in [1, 5)$
 $\Rightarrow f(x) = 0$ at $x = 1 \Rightarrow a = 5$
 $f(x) - 14 = (x - 1)^2 (x - 7)$
 $\frac{f(x) - 14}{(x - 1)^2} = x - 7$



$$= \int_{-1}^0 (x^2 + x + 1) dx + \int_0^1 [(x^2 + x + 1) - 3x] dx$$

$$= \frac{7}{6}$$

8.
$$\int_0^1 0 dx + \int_1^2 (x - 1) dx + \int_2^3 2(x - 2) dx + \int_3^4 3(x - 3) dx + \int_4^5 4(x - 4) dx$$

$$= 5.00$$

9. Here $C_1 C_2 = \sqrt{47}$, $r_1 + r_2 = 8$ and $|r_1 - r_2| = 2$
 $\Rightarrow |r_1 - r_2| < C_1 C_2 < r_1 + r_2 \Rightarrow$ two common tangents.

10. Normal at $P(t_1^2, 2t_1)$ on the parabola $y^2 = 4x$... (i)
 Meets it again at the point $Q(t_2^2, 2t_2)$,
 where $t_2 = -t_1 - \frac{2}{t_1}$... (ii)

If subtends a right angle at the vertex $(0, 0)$ then
 (Slope of OP) (Slope of OQ) = -1
 $\Rightarrow \frac{2t_1}{t_1^2} \cdot \frac{2t_2}{t_2^2} = -1 \Rightarrow t_2 = \frac{-4}{t_1}$... (iii)

From (ii) and (iii), $-t_1 - \frac{2}{t_1} = \frac{-4}{t_1}$
 $\Rightarrow -t_1 = -\frac{2}{t_1}$
 $\Rightarrow t_1^2 = 2 \Rightarrow t_1 = \pm\sqrt{2} \therefore t_2 = \mp 2\sqrt{2}$
 $\therefore P$ and Q are $(2, \pm 2)$ and $(8, \mp 4\sqrt{2})$
 $\therefore PQ = \sqrt{(8 - 2)^2 + (\mp 4\sqrt{2} \mp 2\sqrt{2})^2} = \sqrt{36 + 72}$
 $= \sqrt{108} = 6\sqrt{3}$.